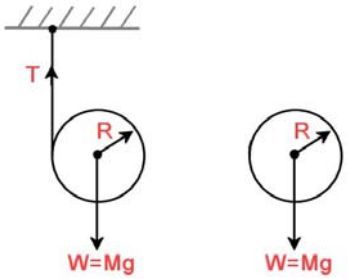
Equation solution for the toilet paper problem!



The downward acceleration a_f of the freely falling toilet paper roll (f):

$$a_f = \frac{W}{M} = \frac{Mg}{M} = g$$

The downward acceleration a_u of the unrolling toilet paper (u):

$$a_u = \frac{W-T}{M} = g - \frac{T}{M}$$

The equation for angular acceleration of the unrolling toilet paper:

$$\alpha = \frac{\tau}{I} = \frac{R\,T}{\gamma\,M\,R^2} = \frac{T}{\gamma MR}$$

where τ is the torque causing the toilet paper to unroll, I is its moment of inertia, and γ is the ratio of the actual moment of inertia I of the roll to MR^2 (a number between ½ and 1).

The constraint between angular and linear acceleration of the unrolling toilet paper:

$$a_u = \alpha R \quad \text{or} \quad \alpha = \frac{a_u}{R}$$

Combining these equations and eliminating α:

$$a_u = \frac{g}{1+\gamma} < g \quad \text{Q.E.D.}$$